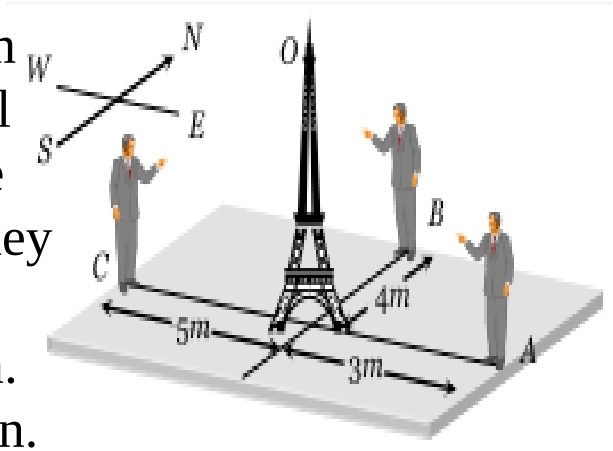
Kinematics of a Particle

To analyse the motion of a particle without knowing why the particles moves, The role of four important quantities :Position, displacement, velocity & acceleration must be clearly understood.

Position



Any object is situated at point O and three observers from three different places are looking for same object, then all three observers will have different observations about the position of point O and no one will be wrong. Because they are observing the object from their different positions.
Observer 'A' says : Point O is 3 m away in west direction.
Observer 'B' says : Point O is 4 m away in south direction.
Observer 'C' says : Point O is 5 m away in east direction.
Therefore position of any point is completely expressed by two factors:
Its distance from the observer and its direction with respect to observer.
That is why position is characterized by a vector known as position vector.

Position vector

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$ **(two dimensional)**

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$ **(three dimensional)**

Rest and Motion

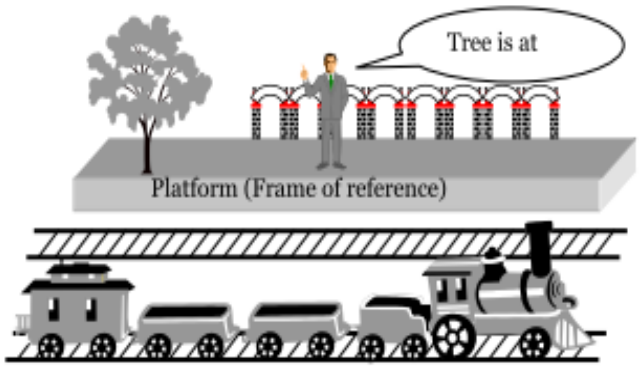
If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

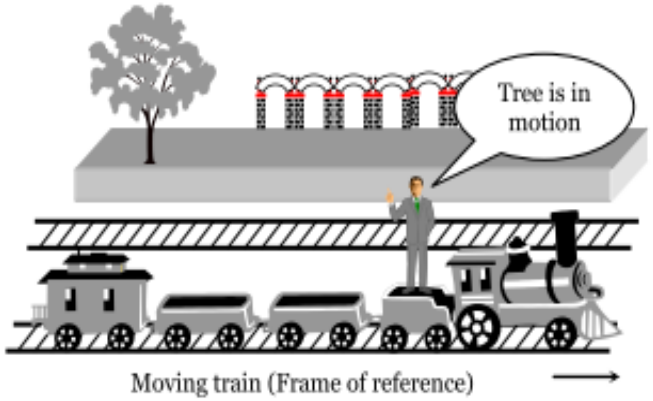
And *if a body changes its position as time passes with respect to frame of reference, it is said to be in motion*.

Frame of Reference : It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

A passenger standing on platform observes that tree on a platform is at rest. But when the same passenger is passing away in a train through station, observes that tree is in motion. In both conditions observer is right. But observations are different because in first situation observer stands on a platform, which is reference frame at rest and in second situation observer moving in train, which is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of references.



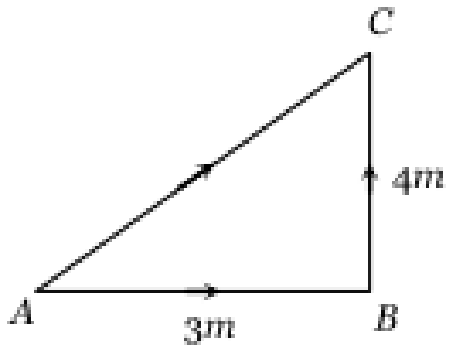


Particle or Point Mass

The smallest part of matter with zero dimension which can be described by its mass and position is defined as a particle.
If the size of a body is negligible in comparison to its range of motion then that body is known as a particle.
A body(Group of particles) to be known as a particle depends upon types of motion. For example in a planetary motion around the sun the different planets can be presumed to be the particles.
In above consideration when we treat body as particle, all parts of the body undergo same displacement and have same velocity and acceleration.

Distance

It is the actual path length covered by a moving particle in a given interval of time.
Distance is a scalar quantity.
Dimension : $[M^0 L^1 T^0]$
Unit : metre (S.I.)
If a particle starts from A and reach to C through point B as shown in the figure. Then distance travelled by particle is AB+BC= 3+4= 7m.

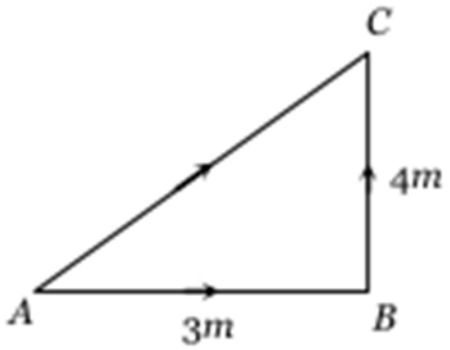


Displacement

Displacement is the change in position vector i.e., A vector joining initial to final position. Displacement is a vector quantity

Dimension : $[M^0 L^1 T^0]$

Unit : metre (S.I.)



In the above figure the displacement of the particle

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\left|\overrightarrow{AC}\right| = \sqrt{AB^2 + BC^2 + 2AB\ BC \cos 90}$

$= 5\text{m}$

Comparison between distance and displacement

(i) The magnitude of displacement is equal to minimum possible distance between two positions. So distance $\geq$ |Displacement|.

(ii) For a moving particle distance can never be negative or zero while displacement can be. (zero displacement means that body after motion has came back to initial position)

i.e., Distance > 0 but Displacement > = or < 0

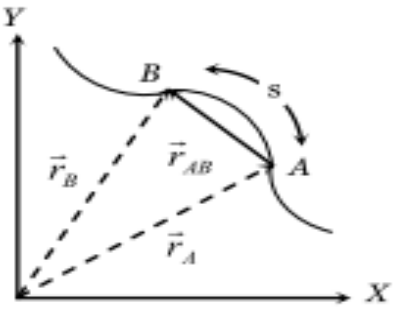
(iii) For motion between two points displacement is single valued while distance depends on actual path and so can have many values.

(iv)For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

(v)In general magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.

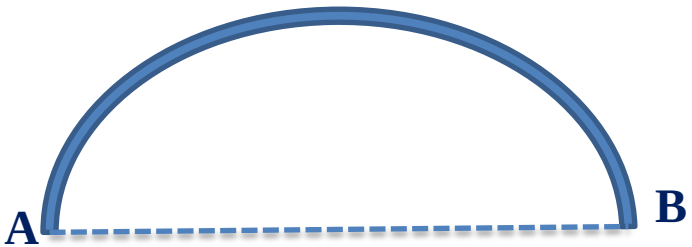
(vi) If $\overrightarrow{r_A}$ and $\overrightarrow{r_B}$ are the position vectors of particle initially and finally.

Then displacement of the particle $\overrightarrow{r_{AB}} = \overrightarrow{r_B} - \overrightarrow{r_A}$ and s is the distance travelled if the particle has gone through the path APB



Exercise on Distance and Displacement

1. A man goes 10m towards North, then 20m towards east then Find the displacement of the man? (Ans. $10\sqrt{5}\ m$)
2. Find the ration of the distance to displacement of the following figure 1.(Ans. $\frac{\pi}{2}$)
3. Consider a cube, where the distance of a boy is shown in the figure 2. Find the shortest distance between A and B? (Ans. $a\sqrt{3}$)
4. A body moves over one fourth of a circular arc in a circle of radius r. Find the magnitude of distance travelled and displacement.(Ans. $\frac{\pi r}{2}$, $r\sqrt{2}$)
5. The displacement of the point of the wheel initially in contact with the ground, when the wheel roles forward half a revolution will be (radius of the wheel is R) (Ans. R $\sqrt{\pi^2 + 4}$)



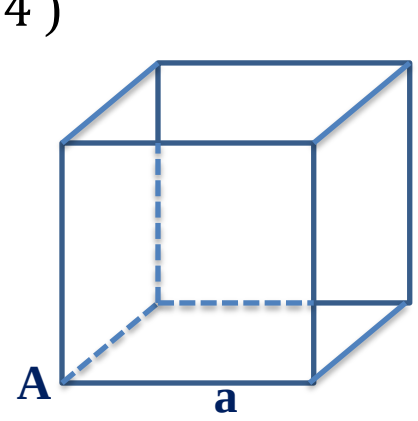


figure 2

Speed , Velocity and acceleration

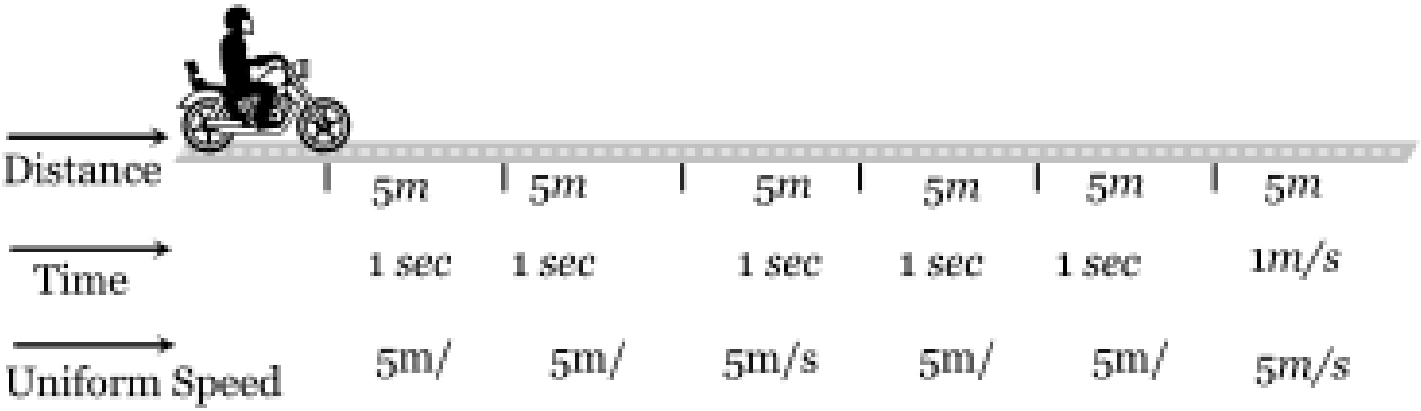
Rate of distance covered with time is called speed. It is a scalar quantity having symbol v .

Dimension : $[M^0 L^1 T^{-1}]$

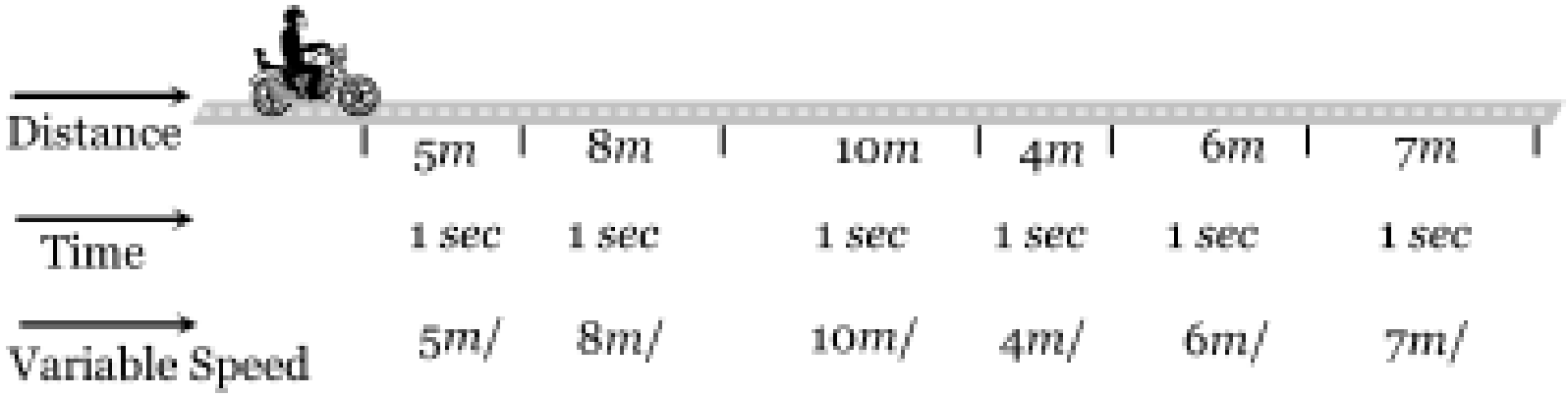
(iii) Unit : m/s (S.I.), cm/s (C.G.S.)

Types of speed

(a) Uniform speed: When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5m) in each second. So we can say that particle is moving with uniform speed of 5 m/s.



Non-uniform (variable) speed : In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5m in 1st second, 8m in 2nd second, 10m in 3rd second, 4m in 4th second etc. Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.



Average speed : The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

Time average speed : When particle moves with different uniform speed v_1,v_2,v_3,…... etc. in different time intervals t_1,t_2,t_3..... etc. respectively, its average speed over the total time of journey is given as

$$v_{avg} = \frac{total\ distance\ covered}{total\ time} = \frac{d_1 + d_2 + d_3 + \cdots}{t_1 + t_2 + t_3 + \cdots} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \cdots}{t_1 + t_2 + t_3 + \cdots}$$

Distance averaged speed : When a particle describes different distances d_1,d_2,d_3,…... with different time intervals t_1,t_2,t_3..... etc. with speeds v_1,v_2,v_3,…... respectively then the speed of particle averaged over the total distance can be given as

$$v_{avg} = \frac{total\ distance\ covered}{total\ time} = \frac{d_1 + d_2 + d_3 + \cdots}{t_1 + t_2 + t_3 + \cdots} = \frac{d_1 + d_2 + d_3 + \cdots}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3} + \cdots}$$

Instantaneous speed : It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (i.e., $\Delta t \to 0$). Thus Instantaneous speed $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

Velocity: *Rate of change of position i.e. rate of displacement with time is called velocity.* It is a vector quantity having symbol $\vec{v}$.

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : metre/second (S.I.), cm/second (C.G.S.)

Types of velocity

Uniform velocity : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

Non-uniform velocity: A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

Average velocity : It is defined as the ratio of displacement to time taken by the body

$$\overrightarrow{v_{avg}} = \frac{total\ displacement}{total\ time} = \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous velocity : Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Acceleration

The time rate of change of velocity of an object is called acceleration of the object. It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)

- There are three possible ways by which change in velocity may occur

When only direction of velocity changes	**When only magnitude of velocity changes**	**When both magnitude and direction of velocity changes**
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
e.g. Uniform circular motion	e.g. Motion under gravity	e.g. Projectile motion

Dimension : $[M^0 L^1 T^{-2}]$

(4) Unit : metre/second ^{2} (S.I.); cm/second ^{2} (C.G.S.)

Types of acceleration :

Uniform acceleration : A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. e.g. Projectile motion.

Non-uniform acceleration: A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

Average acceleration : $\overrightarrow{a_{avg}} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\overrightarrow{v_2} - \overrightarrow{v_1}}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector.

Instantaneous acceleration $\vec{a} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{d\vec{v}}{dt}$

Exercise on speed, velocity and acceleration

1. If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then find average speed. (Ans. $\frac{5v_1v_2}{3v_1+2v_2}$)
2. The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in meters and t in sec. Find the displacement, when velocity is zero. (Ans. 0)
3. The motion of a particle is described by the equation $x = a + bt^2$, where a=15cm and b=3cm. Find its instantaneous velocity at time 3 sec. (Ans. 18cm/s)
4. A person completes half of its his journey with speed v_1 and rest half with speed v_2. Find the average speed of the person. (Ans. $\frac{2v_1v_2}{v_1+v_2}$)
5. Given $S=t^3-6t^2$, Find velocity at t=1 sec and 2sec.
6. Given $S=3t^2 + 4t + 9$, *Find velocity at t=2 sec and avg.velocity at t=1 sec to t=4 sec. (Ans. 16 m/s, 57/3 m/s)*
7. *Given* $\vec{S} = t^2\hat{\imath} + (3t - 1)\hat{\jmath}$ *,* find velocity and acceleration at t= 1 sec? (Ans. $\vec{v} = 2\hat{\imath} + 3\hat{\jmath}$, $\vec{a} = 2\hat{\imath}$)

8. The relation between time t and distance x is $t = \alpha x^2 + \beta x$ where α and β are constants. Find the retardation? (Ans. $2\alpha v^3$)

9. A particle is moving eastwards with velocity of 5 m/s. In 10 sec the velocity changes to 5 m/s northwards. Find average acceleration in this time. Ans. $\frac{1}{\sqrt{2}}$ m/s^2.